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September 9 – 14, 2019, Belgrade, Serbia

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B. Dragovich, I. Salom and M. Vojinović

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P R E F A C E

This volume contains some reviews and original research contributions, which are related to the **10th Mathematical Physics Meeting: School and Conference on Modern Mathematical Physics**, organized by the Institute of Physics, Belgrade (Serbia), September 9–14, 2019. The programme of this meeting was mainly oriented towards some recent developments in gravity and cosmology, string and quantum field theory, and some relevant mathematical methods. We hope that articles presented here will be valuable literature not only for the participants of this meeting but also for many other PhD students and researchers in modern mathematical and theoretical physics. We are grateful to all authors for writing their contributions for these proceedings.

The previous nine meetings in this series of schools and conferences on modern mathematical physics were also held in Serbia: Sokobanja 2001, Kopaonik 2002, Zlatibor 2004, Belgrade 2006, 2008, 2010, 2012, 2014, and 2017. The corresponding proceedings of all these meetings were published by the Institute of Physics Belgrade, and are available in the printed form as well as online at the websites. According to an agreement with the journal Symmetry, several papers are published in the special issue "Selected Papers: 10th Mathematical Physics Meeting".

This jubilary tenth meeting took place at two different venues — the opening and the first day of lectures was held in the grand lecture hall of the Serbian Academy of Sciences and Arts, while the lectures for the remaining five days were held at the Mathematical Institute. Both venues are located in Belgrade downtown, across the road of each other. We hope that all attendees of this meeting will recall it as a useful and pleasant event, and will wish to participate again in the future.

We wish to thank all lecturers and other speakers for their interesting and valuable talks. We also thank all participants for their active participation. Financial support of our sponsors, *Ministry of Education, Science and Technological Development of the Republic of Serbia, Belgrade; Telekom Srbija; Open access journal "Symmetry"*, and the support of our media partner, *Open access journal "Entropy"*, were very significant for realization of this activity.

April 2020

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Review and Research Works

Non-Riemannian volume elements dynamically generate inflation

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Abstract

Our primary objective is the formulation of a plausible cosmological inflationary model entirely in terms of a pure modified gravity without any *a priori* matter couplings within the formalism of non-Riemannian spacetime volume elements. The non-Riemannian volume elements *dynamically* create in the physical Einstein frame a canonical scalar matter field and produce a non-trivial inflationary scalar potential with a large flat region and a low-lying stable minimum corresponding to the late universe stage. This dynamically generated inflationary potential is a substantial generalization of the classic Starobinsky potential. Our model predicts scalar power spectral index and tensor to scalar ratio in accordance with the available observational data.

1. Introduction

The theoretical framework based on the concept of "inflation" in the study of the evolution of the early Universe provides an attractive solution ex-

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plaining the “puzzles” of Big-Bang cosmology (the horizon problem, the flatness problem, the magnetic monople problem, etc. [1]-[5]. Likewise it is an important instrumentarium for treatment of primordial density perturbations [6, 7]. For some recent detailed accounts, see Refs.[8]-[12].

On the other hand, in a parallel development another groundbreaking concept emerged in the last decade or so about the intrinsic necessity to modify (extend) gravity theories beyond the scope of standard Einstein’s general relativity with the main motivation to overcome the limitations of the latter coming from: (i) Cosmology – for solving the problems of dark energy and dark matter and explaining the large scale structure of the Universe [13, 14]; (ii) Quantum field theory in curved spacetime – due to renormalization of ultraviolet divergences in higher loops [15]; (iii) Modern string theory – due to the natural appearance of higher-order curvature invariants and scalar-tensor couplings in low-energy effective field theories [16].

Various classes of modified gravity theories have been employed to construct viable inflationary models: $f(R)$-gravity; scalar-tensor gravity; Gauss-Bonnet gravity (see Refs.[17]-[21] for a detailed accounts); recently also based on non-local gravity (Ref.[22] and references therein) or based on brane-world scenarios (Ref.[23] and references therein). The first early successful cosmological model based on the extended $f(R) = R + R^2$-gravity is the classical Starobinsky potential [1].

A further specific broad class of actively developed modified (extended) gravitational theories is based on the formalism of *non-Riemannian space-time volume elements* (originally proposed in Refs.[24]-[28]; see Refs.[29, 30] for a systematic geometric formulation). This formalism was used as a basis for constructing a series of extended gravity-matter models describing unified dark energy and dark matter scenario [31, 32], quintessential cosmological models with gravity-assisted and inflaton-assisted dynamical suppression (in the “early” universe) or dynamical generation (in the post-inflationary universe) of electroweak spontaneous symmetry breaking and charge confinement [33]-[35], as well as a novel mechanism for dynamical supersymmetric Brout-Englert-Higgs effect in supergravity [29].

In what follows we will describe in some detail the construction of a viable cosmological inflationary model starting from a modified pure gravity involving several independent non-Riemannian volume elements and *without any a priori coupling* to matter fields.

2. Brief Reminder on Non-Riemannian Volume-Forms (Volume Elements)

Let us briefly recall the essence of the non-Riemannian volume-form formalism (cf. Ref.[36]).

In integrals over differentiable manifolds (not necessarily Riemannian one, so *no* metric is needed) volume-forms are given by nonsingular maximal rank differential forms ω:

$$\int_{\mathcal{M}} \omega(\ldots) = \int_{\mathcal{M}} dx^D\, \Omega(\ldots) \;\; , \;\; \omega = \frac{1}{D!}\omega_{\mu_1\ldots\mu_D} dx^{\mu_1} \wedge \ldots \wedge dx^{\mu_D} \; , \tag{1}$$

where $\omega_{\mu_1\ldots\mu_D} = -\varepsilon_{\mu_1\ldots\mu_D}\Omega$ and Ω is the volume element density. Our conventions for the totally anti-symmetric symbols are

$$\varepsilon^{01\ldots D-1} = 1 \; , \; \varepsilon_{01\ldots D-1} = -1.$$

In Riemannian D-dimensional spacetime manifolds a standard generally-covariant volume-form is defined through the "D-bein" (frame-bundle) canonical one-forms $e^A = e^A_\mu dx^\mu$ $(A = 0, \ldots, D-1)$:

$$\omega = e^0 \wedge \ldots \wedge e^{D-1} = \det \| e^A_\mu \| \, dx^{\mu_1} \wedge \ldots \wedge dx^{\mu_D} \; , \tag{2}$$

where the standard Riemannian volume element density reads

$$\Omega = \det \| e^A_\mu \| = \sqrt{- \det \| g_{\mu\nu} \|} \equiv \sqrt{-g}.$$

To construct modified gravitational theories as alternatives to ordinary standard theories in Einstein's general relativity, instead of $\sqrt{-g}$ we can employ one or more alternative *non-Riemannian* volume element(s) as in (1) given by non-singular *exact* D-forms $\omega = dA$, where: $A = \frac{1}{(D-1)!} A_{\mu_1\ldots\mu_{D-1}} dx^{\mu_1} \wedge \ldots \wedge dx^{\mu-1}$ and the corresponding volume element density reads:

$$\Omega \equiv \Phi(A) = \frac{1}{(D-1)!}\varepsilon^{\mu_1\ldots\mu_D}\, \partial_{\mu_1} A_{\mu_2\ldots\mu_D} \; . \tag{3}$$

Thus, non-Riemannian volume element densities $\Phi(A)$ are defined in terms of the (scalar density of the) dual field-strength of auxiliary rank $D-1$ tensor gauge fields $A_{\mu_1\ldots\mu_{D-1}}$.

As an important remark, let us note that in the first-order (Palatini) formalism ($g_{\mu\nu}$ and $\Gamma^\lambda_{\mu\nu}$ *a priori* independent), the auxiliary tensor gauge fields $A_{\mu_1\ldots\mu_{D-1}}$ turn out to be (almost) pure-gauge – *no propagating field degrees of freedom* except for few discrete degrees of freedom with conserved canonical momenta appearing as arbitrary integration constants. See Refs.[30]-[33] (appendices A) for a systematic proof of the latter fact using the standard canonical Hamiltonian treatment of systems with gauge symmetries, *i.e.*, systems with first-class Hamiltonian constraints a'la Dirac [37, 38].

However, in the second-order (metric) formalism (where $\Gamma^\lambda_{\mu\nu}$ is the usual Levi-Civita connection of the metric $g_{\mu\nu}$) the first non-Riemannian volume form $\Phi(A)$, replacing $\sqrt{-g}$ in the modified Einstein-Hilbert part of the action:

$$S = \int d^4x \Phi(A) R + \ldots \; , \tag{4}$$

is *not* any more pure-gauge. The reason is that in the action (4) the scalar curvature R (in the metric formalism) containes *second-order* (time) derivatives (the latter amount to a total derivative in the ordinary case $S = \int d^4x \sqrt{-g} R + \ldots$).

So defining $\chi_1 \equiv \Phi(A)/\sqrt{-g}$, the latter field becomes physical degree of freedom as seen from the equations of motion resulting from varying (4) w.r.t. $g^{\mu\nu}$:

$$R_{\mu\nu} + \frac{1}{\chi_1}\left(g_{\mu\nu}\Box\chi_1 - \nabla_\mu\nabla_\nu\chi_1\right) + \ldots = 0 \tag{5}$$

3. Modified Pure Gravity with Non-Riemannian Volume Elements

Let us now consider the following simple modified gravity model without any couplings to matter fields (we will use "Planck units" $16\pi G_N = 1$):

$$S = \int d^4x \Big\{ \Phi_1(A)\Big[R - 2\Lambda_0 \frac{\Phi_1(A)}{\sqrt{-g}}\Big] + \Phi_2(B)\frac{\Phi_0(C)}{\sqrt{-g}} \Big\} \; . \tag{6}$$

Here R is the scalar curvature in the metric formalism and:

$$\Phi_1(A) \equiv \frac{1}{3!}\varepsilon^{\mu\nu\kappa\lambda}\partial_\mu A_{\nu\kappa\lambda} \; , \; \Phi_2(B) \equiv \frac{1}{3!}\varepsilon^{\mu\nu\kappa\lambda}\partial_\mu B_{\nu\kappa\lambda} \; ,$$
$$\Phi_0(C) \equiv \frac{1}{3!}\varepsilon^{\mu\nu\kappa\lambda}\partial_\mu C_{\nu\kappa\lambda} \; , \tag{7}$$

denote three different independent non-Riemannian volume element densities as in (3) for $D = 4$. Λ_0 is dimensionful parameter which will turn out in what follows to play the role of an inflationary scale.

It is important to stress that the form of the action (6) is uniquely specified by the requirement about global Weyl-scale invariance under:

$$g_{\mu\nu} \to \lambda g_{\mu\nu} \; , \; A_{\mu\nu\kappa} \to \lambda A_{\mu\nu\kappa} \; , \; B_{\mu\nu\kappa} \to \lambda^2 B_{\mu\nu\kappa} \; , \; C_{\mu\nu\kappa} \to C_{\mu\nu\kappa} \; . \tag{8}$$

where $\lambda =$ const. Its importance within the context of non-Riemannian volume element formalism has been originally stressed in [26].

The equations of motion from the action (6) w.r.t. the auxiliary gauge fields $A_{\mu\nu\lambda}$, $B_{\mu\nu\lambda}$, $C_{\mu\nu\lambda}$ defining the non-Riemannian volume elements (7) yield, respectively:

$$R - 4\Lambda_0 \frac{\Phi_1(A)}{\sqrt{-g}} = -M_1 \equiv \text{const} \; , \tag{9}$$

$$\frac{\Phi_0(C)}{\sqrt{-g}} = -M_2 \equiv \text{const} \;\; , \;\; \frac{\Phi_2(B)}{\sqrt{-g}} = \chi_2 \equiv \text{const} \; . \tag{10}$$

Here M_1, M_2 and χ_2 are (dimensionful and dimensionless, respectively) free integration constants; M_1, M_2 indicate spontaneous breaking of global Weyl symmetry (8).

Also, it is important to observe that, since the scalar curvature R contains terms with second-order time derivatives on $g_{\mu\nu}$, Eq.(9) is a genuine dynamical equation of motion and *not* a constraint.

The equations of motion w.r.t. $g_{\mu\nu}$ from (6) read:

$$R_{\mu\nu} - \Lambda_0 \chi_1 \, g_{\mu\nu} + \frac{1}{\chi_1}\Big(g_{\mu\nu}\Box\chi_1 - \nabla_\mu\nabla_\nu\chi_1\Big) - \frac{\chi_2 M_2}{\chi_1} g_{\mu\nu} = 0 \; , \tag{11}$$

with $\chi_1 \equiv \Phi(A)/\sqrt{-g}$. Taking the trace of (11):

$$3\frac{\Box\chi_1}{\chi_1} - \frac{4\chi_2 M_2}{\chi_1} - M_1 = 0 \tag{12}$$

yields a dynamical equation of motion for the composite scalar field χ_1.

4. From Modified Gravity to the Physical Einstein Frame

We now transform Eqs.(11) and (12) to the physical Einstein frame via the conformal transformation $\bar{g}_{\mu\nu} = \chi_1 g_{\mu\nu}$, upon using the well-known (cf. Ref.[39]) conformal transformation formulas (bars indicate magnitudes in the $\bar{g}_{\mu\nu}$-frame):

$$\begin{aligned} R_{\mu\nu}(g) = R_{\mu\nu}(\bar{g}) - 3\frac{\bar{g}_{\mu\nu}}{\chi_1}\bar{g}^{\kappa\lambda}\partial_\kappa\chi_1^{1/2}\partial_\lambda\chi_1^{1/2} \\ + \chi_1^{-1/2}(\bar{\nabla}_\mu\bar{\nabla}_\nu\chi_1^{1/2} + \bar{g}_{\mu\nu}\bar{\Box}\chi_1^{1/2}) \; , \end{aligned} \tag{13}$$

$$\Box\chi_1 = \chi_1\Big(\bar{\Box}\chi_1 - 2\bar{g}^{\mu\nu}\frac{\partial_\mu\chi_1^{1/2}\partial_\nu\chi_1}{\chi_1^{1/2}}\Big) \; . \tag{14}$$

Hereby the transformed equations acquire the standard form of Einstein equations w.r.t. the new "Einstein-frame" metric $\bar{g}_{\mu\nu}$:

$$R_{\mu\nu}(\bar{g}) - \frac{1}{2}\bar{g}_{\mu\nu}R(\bar{g}) = \frac{1}{2}\Big[\partial_\mu u\partial_\nu u - \bar{g}_{\mu\nu}\big(\frac{1}{2}\bar{g}^{\kappa\lambda}\partial_\kappa u\partial_\lambda u + U_{\rm eff}(u)\big)\Big] \; , \tag{15}$$

$$\bar{\Box}u + \frac{\partial U_{\rm eff}}{\partial u} = 0 \; , \tag{16}$$

where we have redefined

$$\Phi_1(A)/\sqrt{-g} \equiv \chi_1 = \exp{(u/\sqrt{3})} \tag{17}$$

in order to obtain a canonically normalized kinetic term for the scalar field u, and where we have obtained a *dynamically generated effective scalar potential*:

$$U_{\rm eff}(u) = 2\Lambda_0 - M_1\exp{(-\frac{u}{\sqrt{3}})} + \chi_2 M_2 \exp{(-2\frac{u}{\sqrt{3}})} \; . \tag{18}$$

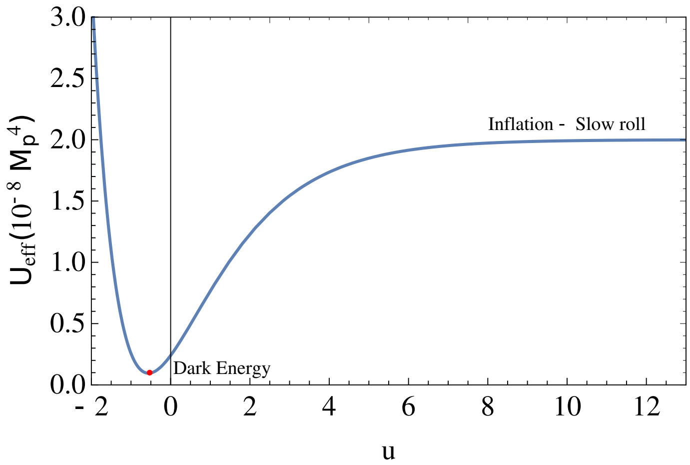

Figure 1: Qualitative shape of the dynamically generated effective scalar potential U_{eff} (18) as function of u. The unit for u is $M_{Planck}/\sqrt{2}$.

U_{eff} (18) is a generalization of the classic *Starobinsky potential* [1]; the latter is a special case of (18) for $\Lambda_0 = \frac{1}{4}M_1 = \frac{1}{2}\chi_2 M_2$.

Accordingly, the corresponding Einstein-frame action reads:

$$S_{\mathrm{EF}} = \int d^4x \sqrt{-\bar{g}} \Big[R(\bar{g}) - \frac{1}{2}\bar{g}^{\mu\nu}\partial_\mu u \partial_\nu u - U_{\mathrm{eff}}(u) \Big] \, , \tag{19}$$

with U_{eff} as in (18).

Let us particularly emphasize that the Einstein-frame action (19) is entirely dynamically generated:

(a) The canonical scalar field u is dynamically created from the ratio of the volume-element densities $\Phi_1(A)/\sqrt{-g}$ (17);

(b) The effective potential $U_{\mathrm{eff}}(u)$ (18) is dynamically generated due to the appearance of the free integration constants $M_{1,2}, \chi_2$ in (18) as a result of the specific (constrained) dynamics (9)-(10) of the auxiliary gauge fields $A_{\mu\nu\lambda}, B_{\mu\nu\lambda}, C_{\mu\nu\lambda}$ – constituents of the non-Riemannian volume element densities $\Phi(A), \Phi(B), \Phi(C)$ (7). $U_{\mathrm{eff}}(u)$ (18) is graphically depicted on Fig.1.

The dynamically generated potential $U_{\mathrm{eff}}(u)$ (18) has two main features relevant for cosmological applications.

First, $U_{\mathrm{eff}}(u)$ (18) possesses a long flat region for large positive u and, second, it has a stable minimum for a small finite value $u = u_*$:

- (i) $U_{\mathrm{eff}}(u) \simeq 2\Lambda_0$ for large u;

- (ii) $\frac{\partial U_{\rm eff}}{\partial u} = 0$ for $u \equiv u_*$ where:

$$\exp(-\frac{u_*}{\sqrt{3}}) = \frac{M_1}{2\chi_2 M_2} \quad , \quad \frac{\partial^2 U_{\rm eff}}{\partial u^2}\Bigr|_{u=u_*} = \frac{M_1^2}{6\chi_2 M_2} > 0 \; . \tag{20}$$

The flat region of $U_{\rm eff}(u)$ for large positive u correspond to "early" universe' slow-roll inflationary evolution with energy scale $2\Lambda_0$. On the other hand, the region around the stable minimum at $u = u_*$ (20) correspond to "late" universe' evolution where:

$$U_{\rm eff}(u_*) = 2\Lambda_0 - \frac{M_1^2}{4\chi_2 M_2} \equiv 2\Lambda_{\rm DE} \tag{21}$$

is the dark energy density value dynamically generated through the free integration constants $M_{1,2}$, χ_2.

5. FLRW Reduction and Evolution of the Homogeneous Solution

Let us mow consider the reduction of the Einstein-frame action (19) to the Friedmann-Lemaitre-Robertson-Walker (FLRW) setting with metric $ds^2 = -N^2 dt^2 + a(t)^2 d\vec{x}^2$, and with $u = u(t)$.

The FLRW-reduced action reads:

$$S_{\rm FLRW} = \int d^4x \Bigl[-6\frac{a\,\dot{a}^2}{N} + Na^3\Bigl(\frac{1}{2}\,\dot{u}^2 + M_1 e^{-u/\sqrt{3}} - M_2\chi_2 e^{-2u/\sqrt{3}} - 2\Lambda_0\Bigr)\Bigr] \; . \tag{22}$$

We will study the evolution of $u = u(t)$ and $a = a(t)$ specified by (22) using the method of autonomous dynamical systems.

The pertinent Friedmann and u-field equations resulting from (22) are given by:

$$H^2 = \frac{1}{6}\rho \;\; , \;\; \rho = \frac{1}{2}\,\dot{u}^2 + U_{\rm eff}(u) \; , \tag{23}$$

$$\dot{H} = -\frac{1}{4}(\rho + p) \;\; , \;\; p = \frac{1}{2}\,\dot{u}^2 - U_{\rm eff}(u) \; , \tag{24}$$

$$\ddot{u} + 3H\,\dot{u} + \frac{\partial U_{\rm eff}}{\partial u} = 0 \; . \tag{25}$$

It is instructive (following Ref.[40]) to rewite the system of Eqs.(23)-(25) in terms of a set of dimensionless variables:

$$x := \frac{\dot{u}}{\sqrt{12}H}, \quad y := \frac{\sqrt{U_{\rm eff}(u) - 2\Lambda_{\rm DE}}}{\sqrt{6}H}, \quad z := \frac{\sqrt{\Lambda_{\rm DE}}}{\sqrt{3}H} \; , \tag{26}$$

with $L_{\rm DE} = \Lambda_0 - \frac{M_1^2}{8\chi_2 M_2}$ as in (21).

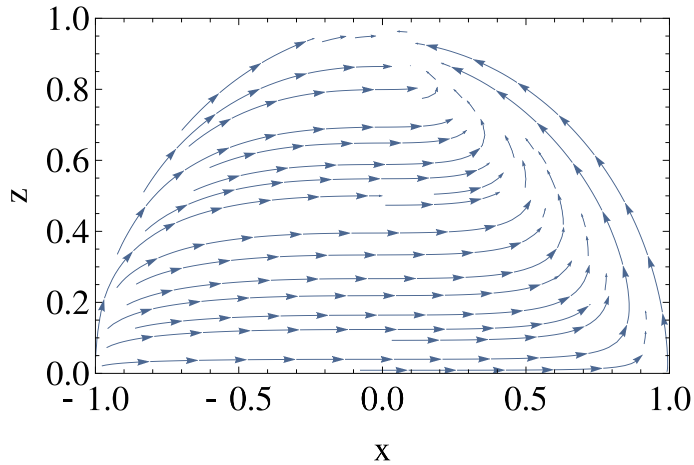

Figure 2: Phase space portrait of the autonomous system (27). The x axis denotes the relative kinetic part of the scalar inflaton, and the z axis denotes the relative part of the dark energy density Λ_{DE}.

The first Friedman Eq.(23) yields an algebraic constraint $x^2+y^2+z^2=1$, so that the autonomous dynamical system w.r.t. (x,z) reads:

$$x' = \frac{\sqrt{3}}{2\Lambda_{DE}} z^2 \left[-M_1\xi(x,z) + 2M_2\chi_2\xi^2(x,z)\right] - 3x(1-x^2) \; , \qquad z' = 3zx^2 \quad , \tag{27}$$

where the primes denote derivative w.r.t. the parameter $\mathcal{N} = \log a$ (number of e-foldings), and the function $\xi(x,z)$ is defined as:

$$\xi(x,z) = \frac{M_1}{2\chi_2 M_2}\Big[1 - \sqrt{\frac{8\Lambda_0 M_2\chi_2}{M_1^2}\,\frac{1-x^2-z^2}{z^2}}\Big] \, . \tag{28}$$

Phase space portrait of the autonomous system (27) is depicted numerically on Fig.2.

The autonomous system (27) possesses the following two critical points:

(a) Stable critical point $A\,(x=0, z=1)$ corresponding to the "late" universe de Sitter behavior with a cosmological constant Λ_{DE} (21).

(b) Unstable critical point $B\left(x=0, z=\sqrt{\Lambda_{\mathrm{DE}}/\Lambda_0}\right)$ corresponding to beginning of evolution in the "early" universe at large u. If the evolution starts at any point close to B, then the dynamics drives the system away from B all the way towards the stable point A at late times.

Numerical solutions of the FLRW system (23)-(25) are graphically presented on Fig.3 for the Hubble parameter $H(t)$, and on Fig.4 for the scalar field $u(t)$.

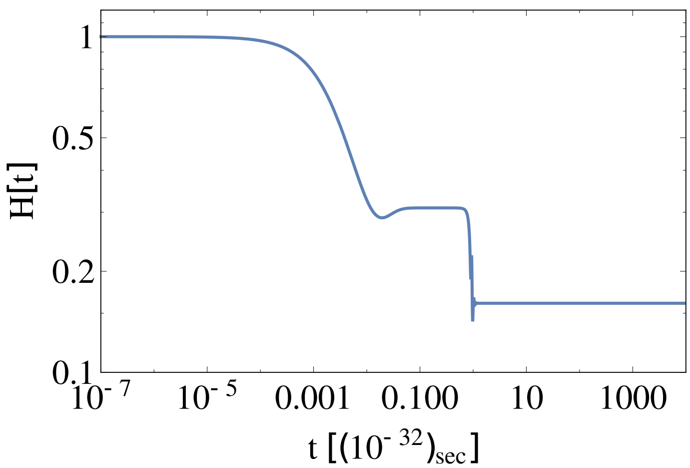

Figure 3: Numerical example of the solution for the Hubble parameter $H(t)$ vs. time. Initially for short times the inflationary Hubble parameter is large and afterwards approaches its cosmological late time value.

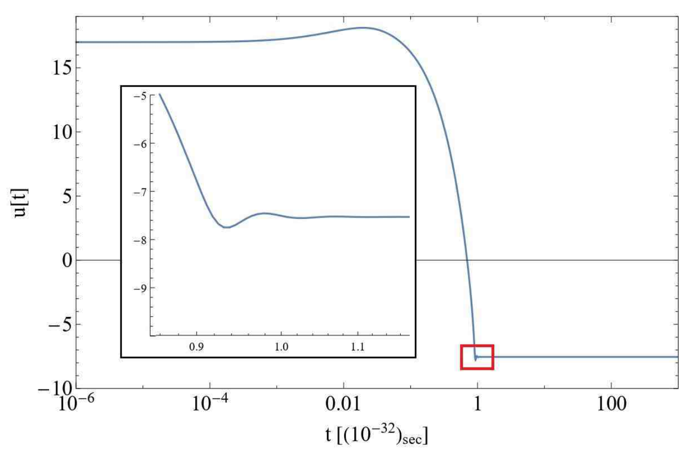

Figure 4: Numerical example of the solution for the scalar field $u(t)$ vs. time. The unit for u is $M_{Planck}/\sqrt{2}$. The blown-up rectangle depicts the oscillations of $u(t)$ around the minimum of U_{eff} (18).

6. Perturbations and Observables

In order to check the viability of our model we will investigate the perturbations of the above FLRW background evolution (23)-(25), in particular focusing on the inflationary observables such as the scalar power spectral index n_s and the tensor-to-scalar ratio r (for definitions, see *e.g.* Ref.[41]). As usual, we introduce the Hubble slow-roll parameters, which in our case using the potential $U_{\rm eff}(u)$ (18) read:

$$\epsilon = \Bigl(\frac{U'_{\rm eff}(u)}{U_{\rm eff}(u)}\Bigr)^2 = \frac{4\zeta^2}{3}\frac{(1/2-\zeta)^2}{\left[(1/2-\zeta)^2+\delta/4\right]^2} \; , \tag{29}$$

$$|\eta| = 2|\frac{U''_{\rm eff}(u)}{U_{\rm eff}(u)}| = \frac{2\zeta}{3}\frac{(1-4\zeta)}{\left[(1/2-\zeta)^2+\delta/4\right]} \; , \tag{30}$$

where:

$$\zeta \equiv \frac{M_2\chi_2}{M_1}\, e^{-u/\sqrt{3}} \quad , \quad \delta \equiv \frac{8M_2\chi_2}{M_1^2}\Lambda_{\rm DE} \; , \tag{31}$$

with $\Lambda_{\rm DE}$ – the dark energy density (21), and therefore the parameter δ being very small.

Inflation ends when $\epsilon(u_f)=1$ for some $u=u_f$ whose value (using the short-hand notation $\zeta_f \equiv \frac{M_2\chi_2}{M_1}e^{-u_f/\sqrt{3}}$) is given by:

$$\zeta_f = \frac{1}{2(1+2/\sqrt{3})}\Bigl[1+\frac{1}{\sqrt{3}}-\sqrt{1/3-(1+2/\sqrt{3})\delta}\Bigr] \simeq \frac{1}{2(1+2/\sqrt{3})} \; . \tag{32}$$

For the number of e-foldings $\mathcal{N} = \frac{1}{2}\int_{u_i}^{u_f} du\, U_{\rm eff}/U'_{\rm eff}$ we obtain:

$$\mathcal{N} = \frac{3}{8}(1+\delta)\Bigl(1/\zeta_i - 1/\zeta_f\Bigr) - \frac{3}{4}(1-\delta)\log\frac{\zeta_f}{\zeta_i} + \frac{3}{4}\delta\,\log\Bigl(\frac{1-2\zeta_i}{1-2\zeta_f}\Bigr) \; , \tag{33}$$

where $\zeta_i \equiv \frac{M_2\chi_2}{M_1}e^{-u_i/\sqrt{3}}$ and $u=u_i$ is very large corresponding to the start of the inflation.

Ignoring the very small δ we have for $\mathcal{N}$ approximately:

$$\mathcal{N} \simeq \frac{3M_1}{8M_2\chi_2}e^{u_i/\sqrt{3}} - \frac{\sqrt{3}}{4}u_i - \frac{3}{4}(1+2/\sqrt{3}) + \frac{3}{4}\log\Bigl(2(1+2/\sqrt{3})\Bigr) \; . \tag{34}$$

Using the slow-roll parameters (29)-(30), one can calculate the values of the scalar spectral index n_s and the tensor-to-scalar ratio r, respectively, as functions of the e-foldings $\mathcal{N}$:

$$r \approx 16\,\epsilon(u_i(\mathcal{N})) \quad , \quad n_s \approx 1 - 6\,\epsilon(u_i(\mathcal{N})) + 2\eta\,(u_i(\mathcal{N})) \; , \tag{35}$$

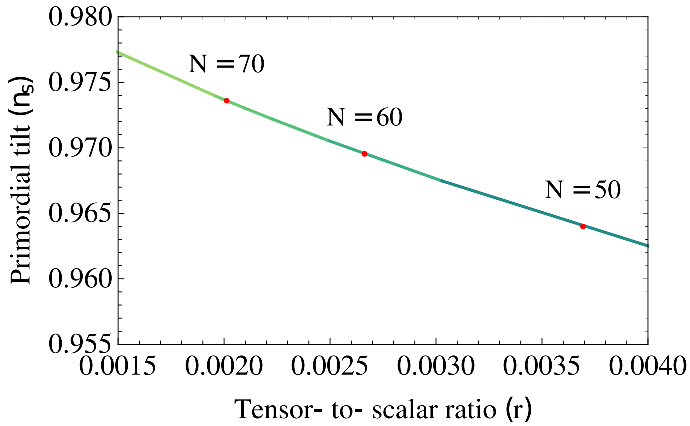

Figure 5: The predicted values of the r and n_s for different number of e-foldings

where $u_i(\mathcal{N})$ is the solution of the transcedental Eq.(34) for u_i as a function of $\mathcal{N}$. From (35), (34), (29), (30) we find:

$$r \simeq \frac{12}{\Big[\mathcal{N} + \frac{\sqrt{3}}{4} u_i(\mathcal{N}) + c_0\Big]^2} \quad , \quad c_0 \equiv \frac{\sqrt{3}}{2} - \frac{3}{4}\log\Big(2(1+2/\sqrt{3})\Big) \; , \tag{36}$$

$$n_s \simeq 1 - \frac{r}{4} - \sqrt{\frac{r}{3}} \quad . \tag{37}$$

The numerical results for (36)-(37) are depicted on Fig.5.

The different values of the r and n_s are compatible with the PLANCK observational data ($0.95 < n_s < 0.97$, $r < 0.064$) (cf. Ref.[42]).

Indeed, for the viable example of $\mathcal{N} = 60$ e-foldings until the end of inflation we obtain from (34)-(37):

$$n_s \approx 0.969 \quad , \quad r \approx 0.002 \; . \tag{38}$$

7. Conclusions

- We proposed a very simple modified gravity model without any initial coupling to matter fields in terms of several alternative non-Riemannian spacetime volume elements within the second order (metric) formalism.
- We show how the non-Riemannian volume elements, when passing to the physical Einstein frame, create a canonical scalar field and produce dynamically a non-trivial inflationary-type potential for the

latter possesing a large flat region describing slow-roll inflation and a stable low-lying minimum corresponding to the late universe stage.

- We study the evolution and stability of the cosmological solutions from the point of view of the theory of dynamical systems. Our model predicts scalar spectral index $n_s \approx 0.969$ and tensor-to-scalar ratio $r \approx 0.002$ for 60 e-folds, which is in accordance with the observational data.

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